(c) Problems of large deviations.

(4) *Study in infinite horizon and asymptotic behaviour.* This contains the classical convergence problem for the sequence (θ_n). It arises naturally when (2) admits one or several asymptotically stable points θ_{*i}. Comments will be made on a row of results [3, 4, 7].

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Estimation Theory and Statistical Physics

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In previous work, I have shown how there are striking similarities between problems of estimation of stochastic processes indexed by time and problems of statistical physics. Thus, with the standard n-dimensional Kalman Filtering Problem, there is an associated Lie algebra of operators which turns out to be the Oscillator Algebra of dimensional at most $2n+2$. This Lie algebra is a solvable Lie algebra which has the n-dimensional Heisenberg algebra as its desired algebra. Constructing the filter corresponds to integrating the Lie algebra. It is natural to extend these ideas to estimation of Random Fields (Gibbs Fields) and conversely to investigate whether problems of statistical physics are amenable to solution via methods of Stochastic System Theory.

In this lecture I discuss various problems in Early Vision using models of Images as Markov Random Fields corrupted by Noise. In theories of vision, it is believed that most problems of early vision are of a computational nature. Examples of such problems are edge detection, surface reconstruction and extraction of depth information from stereo images. We show that all these problems can be formulated in a unified way in the framework of Bayesian Estimation Theory and this also provides the 'correct' method for incorporating a priori information. We discuss optimal estimates and show how the concept of an innovations field plays an important role in these problems.

The models considered in this lecture are:

(i) Gibbs Fields corresponding to 2-dimensional Ising Models corrupted by noise.

(ii) Lattice Approximations to the Free Euclidean Field corrupted by noise.

(iii) Lattice Approximation to the Free Euclidean Field coupled to a Yang–Mills Field observed in noise.

Computing the conditional distribution of the field conditioned by noise gives rise to Gibbs random fields with a random external field.

Understanding the order-disorder phenomena for these systems is a topic of current interest in mathematical physics. The concept of temperature has a natural meaning in the context of estimation of random fields, and the problem of estimation of parameters such as temperature from noisy observations is also discussed.

In the context of these problems we discuss the concept of Stochastic Quantization (as originally proposed by Parisi and Wu) and show its analogy to the relationship between Wiener and Kalman Filtering Problems.

The actual computation of these estimates can be done using Monte Carlo Methods such as the Metropolis Algorithm or Simulated Annealing. Some results on the asymptotic analysis of simulated annealing are presented. Moreover, the solutions of these estimation problems can be implemented in a distributed architecture.

In the final part of this lecture I show how ideas of Scattering Theory as developed by Adamjan and Arov combined with ideas from Stochastic Systems Theory can lead to solutions of problems in Statistical Physics.

Minimizing the Expected Time to Reach a Goal

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The results below were obtained in joint work witn D. Heath, V. Pestien, and W. Sudderth.

Consider the following control problem: On the interval $(0, 1]$ the process $X(t)$ is given by

$$X(t) = x + \int_0^t X(s)[\mu(s)\,\mathrm{d}s + \sigma(s)\,\mathrm{d}W(s)]$$

where W is a standard Wiener process and μ and σ are non-anticipating controls to be chosen subject to $(\mu(t), \sigma(t)) \in \mathcal{S}$, where $\mathcal{S}$ is a specified subset of $\mathbb{R} \times \mathbb{R}_+$, and the object is to minimize the expectation of $T = \inf\{t\colon X(t) = 1\}$. Denoting the desired infimum by $V(x)$ (the value function) it was shown by Pestien and Sudderth [Continuous-time Red and Black: How to control a diffusion to a goal, to appear in Math. of O.R.] that if $M = \sup\{\mu - \sigma^2/2\colon (\mu, \sigma) \in \mathcal{S}\}$ then $V(x) = -(\log x)/M$ and if $M = \mu_0 - \sigma_0^2/2$ for some $(\mu_0, \sigma_0) \in \mathcal{S}$ then $\mu(t) \equiv \mu_0$, $\sigma(t) \equiv \mu_0$, $\sigma(t) \equiv \sigma_0$ is optimal, provided $\lambda\mathcal{S} \subseteq \mathcal{S}$ for $0 \leqslant \lambda < \infty$. Later Heath and Sudderth showed the result remains correct provided $\lambda\mathcal{S} \subseteq \mathcal{S}$ for $0 \leqslant \lambda \leqslant 1$. Here we show the result is correct if and only if $M < \infty$ and $I = \inf_{\varepsilon>0} \sup\{\mu - (\frac{1}{2} - \varepsilon)\sigma^2\colon (\mu, \sigma) \in \mathcal{S}\} < \infty$.

In their original work (cited above) Pestien and Sudderth introduced a very general 'verification theorem' for control problems. This still is insufficient to handle